

What does the Existence & Uniqueness Theorem tell you about the IVP $(\cos x)y' - y^{\frac{3}{2}} = 0$, $y(\frac{\pi}{4}) = 0$?

SCORE: ____ / 3 PTS

Justify your answer properly, but briefly.

$\textcircled{\frac{1}{2}}$ $y' = \frac{y^{\frac{3}{2}}}{\cos x} = f$, so $f_y = \frac{\frac{3}{2}y^{\frac{1}{2}}}{\cos x}$ WHICH IS NOT DEFINED/CONTINUOUS AROUND $(\frac{\pi}{4}, 0)$ WHERE $y < 0$

SO $E+U$ TELLS US NOTHING, $\textcircled{1}$

Consider the DE $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3$.

SCORE: ____ / 6 PTS

[a] Is $y = x^3 + Ax^2 + Bx$ a family of solutions of the DE ?

$$y' = 3x^2 + 2Ax + B$$

$$y'' = 6x + 2A$$

$$x^2 y'' - 2x y' + 2y = \underbrace{x^2(6x + 2A)}_{\textcircled{1}} - \underbrace{2x(3x^2 + 2Ax + B)}_{\textcircled{1}} + \underbrace{2(x^3 + Ax^2 + Bx)}_{\textcircled{1}}$$
$$= \underbrace{2x^3}_{\textcircled{\frac{1}{2}}}$$

NO $\textcircled{\frac{1}{2}}$

[b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions $y(1) = 3$, $y'(1) = -1$.
If the answer to [a] is "NO", skip this part.

MULTIPLE CHOICE. Consider the DEs

SCORE: ____ / 3 PTS

[1] $x''y - y^2 = 2x$

[2] $(5r + 1)\frac{dr}{d\theta} = 2\theta + 1$

[3] $(5w + 1)du + (\ln w - 3u)dw = 0$

(where w is the independent variable)

Which of the DE above are linear? Circle the correct answer below.

(a) none are linear

(b) only [1] is linear

(c) only [2] is linear

(d) only [3] is linear

(e) only [1] & [2] are linear

(f) only [1] & [3] are linear

(g) only [2] & [3] are linear

(h) all are linear

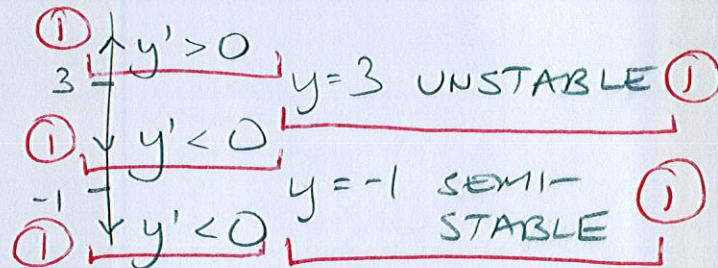
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Consider the autonomous DE $y' = (y+1)^2(y-3)$.

SCORE: ____ / 7 PTS

- [a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.

$$(y+1)^2(y-3)=0 \rightarrow y = -1, 3$$



- [b] If $y = f(x)$ is a solution of the DE such that $f(-4) = 1$, what is $\lim_{x \rightarrow \infty} f(x)$? HINT: Sketch a possible graph of $y = f(x)$.

-1 ①

- [c] If $y = g(x)$ is a solution of the DE such that $g(5) = -2$, what is $\lim_{x \rightarrow \infty} g(x)$?

$-\infty$ ①

FILL IN THE BLANKS.

SCORE: _____ / 3 PTS

[a] The order of the DE $y^{10} - y^7 y^{(4)} = (x^6 + y''')^5$ is 4. i

[b] If $y = \sqrt{x+4}$ is a solution of the DE $y'' = f(x, y, y')$, the largest possible interval of definition is $(-4, \infty)$

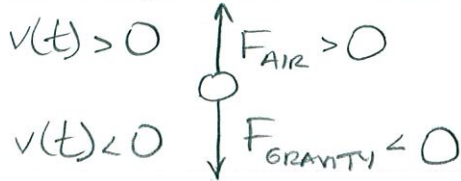
$$y' = \frac{1}{2}(x+4)^{-\frac{1}{2}}$$

$$y'' = -\frac{1}{4}(x+4)^{-\frac{3}{2}}$$

NOTE: -4 IS NOT
IN THE INTERVAL

Write a differential equation for the velocity $v(t)$ of a falling object if the air resistance is proportional to the square of the velocity. Assume that $v(t) > 0$ corresponds to the object moving upward, $v(t) < 0$ corresponds to the object moving downward. (NOTE: This is NOT the same problem as in the homework.)

SCORE: ____ / 3 PTS



$$m \frac{dv}{dt} = -mg + kv^2$$

① ① ①

MUST HAVE
CORRECT SIGN

Consider the IVP $y' = 10x - 5y$, $y(1) = -4$.

SCORE: ____ / 5 PTS

Use Euler's method with $h = 0.2$ to estimate $y(1.4)$.

$$y(1.2) \approx y(1) + y'(1)(0.2) = -4 + (10(1) - 5(-4))(0.2)$$

$$\stackrel{=}{\textcircled{2}} -4 + 30(0.2) = \textcircled{2}$$

$$y(1.4) \approx y(1.2) + y'(1.2)(0.2) \approx \underbrace{2 + 2(0.2)}_{\textcircled{1}} = \underbrace{2.4}_{\textcircled{1}}$$